From Antiquity to 3D Modeling: The Moon Machine

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Abstract
Observing the lunar cycles has been relevant to humankind for millennia, and already in ancient Babylon their fundamental laws were discovered: 19 solar years correspond to almost exactly 235 synodic lunar months or full cycles of moon phases. This so-called Metonic cycle provides a framework for building lunisolar calendars by synchronizing the solar with the lunar year. The Metonic cycle can be visualized using the machine metaphor, and turned into a virtual online and even a mechanical, 3D-printable moon phase calculator.

Keywords
astronomy, moon, lunar phases, metonic cycle, 3d printing, science history

1. Introduction

Across human cultures, observation of the lunar phases has played a central role in timekeeping, providing a natural basis for calendrical systems. Many historical calendars are founded on lunar or lunisolar cycles. Megalithic alignments dating to approximately 4500 BCE were constructed for solar and lunar observation purposes [1], and throughout history a variety of lunar (e.g., Islamic, ancient Roman) and lunisolar (e.g., Babylonian, Chinese, ancient Greek, Jewish) calendars have been in use [2].

The Antikythera Mechanism—an astronomical calculating device attributed to the mathematician and astronomer Poseidonius (135–51 BCE), likely constructed on Rhodes in the 2nd century BCE—demonstrates that ancient Greek scientists were capable of mechanizing the synodic, sidereal, Draconic, and anomalistic lunar cycles [3]. The underlying astronomical principles, including the Metonic cycle [4] and the Draconic period [5], have been known for millennia. Today, advanced interactive visualization techniques and 3D printing technology have become widely accessible. This article shows that these tools enable the construction of lunisolar calendars based on principles derived from ancient astronomy that remain astonishingly accurate to this date.

2. The Metonic cycle

Meton of Athens was a Greek mathematician and astronomer in the fifth century BCE. He is credited with the construction of a large sundial, the remnants of which are still extant. Although none of his written works have survived, Meton—like many scholars of his time—most likely investigated the relationship between solar and lunar timekeeping. He is best known due to the Metonic cycle, named in his honor: a 19-year period that provides a close synchronization of the solar with the lunar year.

The relationship between the solar and lunar cycles is inherently complex. A solar year has a mean duration of approximately 365.242 days, whereas a synodic lunar month—the interval between two successive identical lunar phases—lasts about 29.53 days. Harmonizing these two incommensurate periods represents a fundamental mathematical challenge for calendrical computation.

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This difficulty was addressed through the discovery of the Metonic cycle. As early as the eighth century BCE, Babylonian astronomers recognized that lunar phases (e.g., new or full moon) recur on the same calendar dates at intervals of 19 years. In effect, the Moon returns to nearly the same phase on the same day of the solar year after each 19-year period. For instance, the last full moon of 2025 occurred on 5 December, as it did 19 years earlier, in 2006, and as it will again in 2044.

The Metonic cycle corresponds closely to 235 synodic months and deviates by approximately one day every 218.768 years [6].

$$235 \,^{\text{lunar months}} / 19 \,^{\text{solar years}} = 12.368421053 \,^{\text{lunar months/year}}$$

Despite this fundamental insight, the computation of lunar and solar calendars remained challenging until the 3rd and 2nd centuries BCE, when Greek astronomers and engineers developed the Antikythera Mechanism—an astronomical calculating device capable of mechanically simulating lunar cycles [7] and likely the positions of the planets known to ancient Greek astronomy. The remains of the mechanism were recovered in 1901 by Greek sponge divers, and three principal fragments (out of a total of 82) are on display at the National Archaeological Museum in Athens.

Recent investigations using high-resolution, high-voltage X-ray computed tomography [8] and Polynomial Texture Mapping (PTM) have revealed the functions of the device. The studies confirm that the front dial displayed the sidereal and synodic lunar cycles, while the rear dials represented the Metonic cycle (including the four-year Callippic cycle), the Saros cycle (including the three-year Exeligmos cycle), and the four-year cycle of the Panhellenic Games—the latter constituting the only non-astronomical information processed by the device.

Figure 1: The Antikythera Mechanism (fragment A), front (left) and side view (right), via New York University, Digital Faculty Archives (A. Jones, https://archive.nyu.edu/handle/2451/44191).

Figure 2: The Antikythera Mechanism (virtual reconstruction), front and back faces, Virtual Reality model [9] (own work).

Over recent decades, intensive investigation of the Antikythera Mechanism has demonstrated both its astronomical accuracy and the sophistication of its mechanical design. Several plausible reconstructions propose the existence of an even more advanced subsystem capable of indicating not only the positions of the Sun and the Moon, but also those of Mercury, Venus, Mars, Jupiter, and Saturn—the five planets known to ancient Greek astronomy. Although no physical remnants of this additional gearing have yet been identified, substantial evidence supports its former existence. This evidence derives from references in Latin literature [10], inscriptions on the front and rear plates of the device, physical features of the four-spoked solar gear preserved in fragment A, and the overall architecture of the gear train [11].

Given the remarkable complexity of the Antikythera Mechanism's gearwork, most scholars agree that it must have been preceded by earlier devices, some of which have been attributed to the Greek mathematician and engineer Archimedes of Syracuse (287–212 BCE) [12]. Because the most readily observable lunar cycle is the sequence of lunar phases, it is reasonable to infer that an early form of such a proto-calculator would have incorporated a mechanism for simulating lunar phases based on the Metonic cycle.

In its simplest mechanical realization, this principle could be implemented using a pair of spur gears with a tooth ratio of 235:19. However, this approach entails a very large gearwheel that would have been difficult to manufacture and costly, given the high value of bronze in antiquity. Moreover, such a configuration would not permit coaxial output of the solar-year and lunar-phase indicators, as observed on the front dial of the Antikythera Mechanism.

A more practical solution is to decompose the Metonic ratio into a product of smaller rational fractions, thereby enabling the use of more compact gears while at the same time achieving coaxial output. The Metonic cycle may be split into a product of two smaller rational gear ratios. Because the minimum practical tooth count for a modern involute spur gear is approximately 14—so as to avoid undercutting and to ensure smooth operation—the decomposition

$$235/19 = 47/14 \cdot 70/19 \approx 12.37$$

yields a gearing layout that is both easier to manufacture and capable of providing coaxial output.

A subsequent design choice concerns the method of lunar phase visualization. In the Antikythera Mechanism, this function was realized by a rotating black-and-white sphere mounted on the sidereal lunar pointer, thereby simultaneously indicating the Moon's position relative to the fixed stars and its phase.

A simpler phase-display principle is employed in many modern wristwatches. In this arrangement, the synodic cycle of the Moon is represented by a rotating disc bearing two full-moon symbols that move beneath a fixed aperture. As the disc rotates, the lunar image is progressively revealed and obscured, thereby simulating the waxing and waning phases. After one synodic month, the disc has completed half a rotation, bringing the second full-moon symbol into view and initiating the next cycle. The curved aperture ("bosom"-shaped mask) creates the visual impression of a continuously changing lunar silhouette.

The principle of a rotating disc bearing two lunar images is already attested in the so-called Byzantine Sundial, a portable sundial–calendar preserved in the Science Museum, London. The artifact consists of four surviving parts (including front sundial plate, suspension arm, Moon disc with gear, and arbor with ratchet and two gear wheels). It was purchased in the early 1980s without a clear reported findspot or precise discovery context, and its maker remains unknown. This artifact is generally dated to approximately 400–600 CE. The presence of mathematical gearing in this Byzantine instrument indicates a continuity of the Hellenistic mechanical tradition, as attested by the Antikythera Mechanism, into the Byzantine period and suggests a possible transmission of this knowledge to the Islamic world.

Other than the earlier Antikythera Mechanism, the Byzantine Sundial represents the lunar phase by means of a rotating disc with two inlaid Moon symbols [13]. As in modern wristwatch displays, the lunar disc in this device must rotate at half the angular velocity of the lunar-phase sphere of the Antikythera Mechanism, because it encodes two synodic lunar months within a single full rotation rather than one.

Figure 3: The Byzantine Sundial-Calendar (Moon disc with gear, arbor with ratchet and two gears). [Public Domain], via Science Museum Group (https://www.jstor.org/stable/community.26294717).

If a lunar-phase display employing a double-moon disc of the type used in the Byzantine Sundial is adopted, the mechanical implementation of the Metonic cycle can be further simplified by halving the required maximum tooth count of the 70-tooth gearwheel. This modification reduces both the size of the largest gear and the overall engineering complexity of the gearing system.

$$235/19/2 = 47/14 \cdot 35/19 \approx 6.184$$

Coaxial output can be achieved by appropriate adjustment of the gear modules (i.e., the tooth size, defined as the ratio of the pitch diameter to the number of teeth). The center distance `a` between two meshing gears is determined by their tooth numbers `t`^{1} and `t`^{2} and by the module `m`.

$$a = m \cdot \frac{t^1 + t^2}{2}$$

For the gear train considered here, the required module ratio between the 47/14 and 35/19 gear pairs is ≈ 1:1.123. Using similar module sizes results in a harmonious step ratio, a comparable load distribution across both gear stages, and thus a similar tooth stiffness.
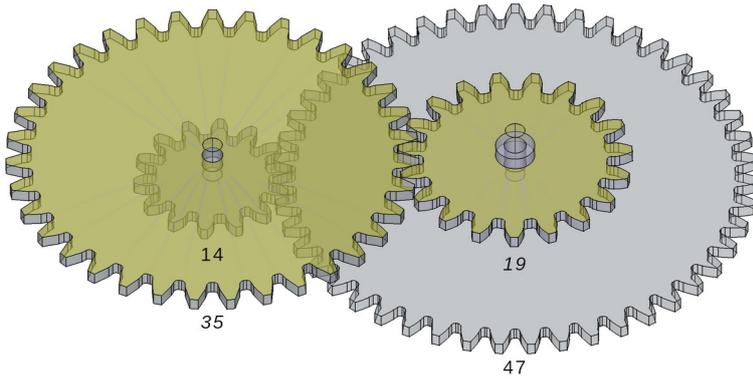
Figure 4: Tooth count determination for a gearing actuating a double-moon disc (own work).

The resulting mechanism, composed of gears with no more than 47 teeth, is therefore more compact and substantially easier to manufacture.

3. Lunar phase calculator models

Our digital project, **"Meton I"**, is a virtual mechanical moon-phase calculator inspired by the Antikythera Mechanism [14]. Using the gear ratios described above, it reproduces the synodic lunar cycle based on the Metonic cycle. A second set of virtual gears, with a

$$4725/352 = 75/16 \, {}^* \, 63/22 \approx 13.42$$

ratio based on recent hypotheses regarding a Draconic gear train in the Antikythera Mechanism [15], simulates the Draconic cycle and tracks the Moon's position relative to the lunar nodes.

The virtual mechanism exhibits a considerable accuracy, with a deviation of approximately one day every ~219 years for the synodic cycle and every ~28 years for the Draconic cycle.
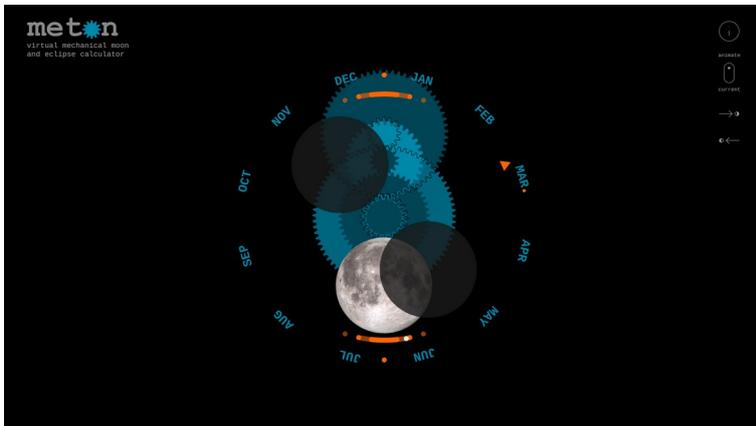

Figure 5: Project "Meton" (screenshot, own work).

The controls on the top right panel enable users to toggle between continuous animation and the current lunar phase, as well as to navigate directly to the preceding or subsequent new or full moon, either retrospectively or prospectively. The calculated date is displayed at the bottom. A small orange marker on the right indicates the vernal equinox on 21 March, which serves as the reference date for Easter calculations. The animation progresses at a rate of one solar year per minute.

"Meton II" was our follow-up project aimed at producing a physical lunar-phase calculator using 3D-printing technology or alternative construction methods [16]. The device features a solid

two-part body housing the gear train, with an aperture at the top for lunar-phase visualization. The mechanism is manually operated via the 35-tooth spur gear, which can be rotated clockwise to explore past dates and counterclockwise for future dates.

In this configuration, the 47-tooth gear drives the solar-year pointer along a calendar scale, while the 19-tooth gear actuates a double-moon disc. The disc incorporates two white moon inlays, partially obscured by an aperture cover, to simulate the full and partial phases of the Moon.

Owing to the dimensional and precision limitations of 3D printing, a base module of 2 mm was selected for the 47/14 gear pair, resulting in a corresponding module of 3.036 for the 35/19 pair.

"Meton II" allows determination of the lunar phase by adjusting the solar-year pointer to a desired date, or conversely, setting the moon disc to a new or full moon to identify the corresponding calendar date. The mechanism comprises a total of 12 fully 3D-printable components. All components are designed without any overhangs—that is, features extending outward without underlying support—and consequently do not require support structures for fabrication. Once assembled, the device measures 9.3 * 12.7 * 1 cm.
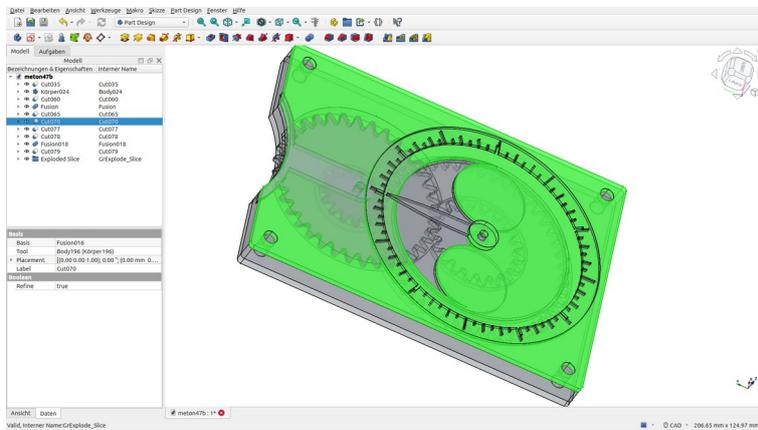

Figure 6: Project "Meton II", CAD view with transparent parts (own work).

Figure 7: Project "Meton II", 3D-printed in black and white PETG (left) and CNC/laser cut in oak wood and steel (right) (own work).

Both the digital calculator **"Meton"** and the 3D models for the physical device **"Meton II"** are released as Open Data. The website source code and all components required to construct the physical mechanism can be freely downloaded and reproduced without any legal restrictions.

4. Conclusion

Visualizing abstract astronomical concepts or complex historical mechanisms presents a significant challenge. Although numerous visual representations exist, they are typically either physical or digital. Physical models are time-consuming and costly to produce and require specialized skills and infrastructure rarely available in most institutions. Digital visualizations, by contrast, are often static and provide limited insight into their true nature and potential; if animated or presented as video, they tend to be linear and largely non-interactive.

This paper demonstrates how archaeoastronomical research on ancient mechanisms, along with their underlying mathematical and mechanical principles, can inform contemporary digital and analog visualizations for both educational and research purposes. The visualizations of the Metonic cycle presented above are both fully dynamic and interactive, facilitating direct, hands-on exploration by their user. The absence of copyright restrictions for the online and physical models ensures unrestricted access for scholars and educators worldwide, facilitating low-barrier engagement with relevant historical concepts and artifacts, thus enabling direct exploration of the remarkable scientific and technological achievements of their time.

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